DEPARTMENT OF THE ARMY
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Technical Letter
No. 1110-2-256

24 June 1981

Engineering and Design
SLIDING STABILITY FOR CONCRETE STRUCTURES

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Engineer Technical
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SLIDING STABILITY FOR CONCRETE STRUCTURES

1. Purpose: This ETL contains criteria and guidance for assessing the sliding stability of gravity dams and other concrete structures.

2. Applicability. This letter is applicable to all field operating activities having civil works design responsibilities.

3. References.

a. ER 1110-2-1806, "Earthquake Design and Analysis for Corps of Engineers Dams."

b. EM 1110-1-1801, "Geological Investigation."

c. EM 1110-2-1803, "Subsurface Investigation-Soils."

d. EM 1110-2-1902, "Stability of Earth and Rockfill Dams."

e. EM 1110-2-1906, "Laboratory Soils Testing."

f. EM 1110-2-1907, "Soil Sampling."

g. EM 1110-2-2200, "Gravity Dam Design."

h. EM 1110-2-2501, "Flood Walls."

i. EM 1110-2-2502, "Retaining Walls."

j. Rock Testing Handbook, "Standard and Recommended Methods," 1978. Available from U.S. Army Waterways Experiment Station, P.O. Box 631, Vicksburg, MS 39180.

k. Henny D.C., "Stability of Straight Concrete Gravity Dams," Transactions, American Society of Civil Engineers, Vol, 99, 1934. Available from Publications Sales Office, Civil Engineering-ASCE, 345 East 47th St., New York, NY 10017.

l. International Society for Rock Mechanics, Commission on Standardization of Laboratory and Field Tests, "Suggested Methods for Determining Shear Strength," Document No. 1, February 1974. Available from Printing and Publishing Office, National Academy of Sciences, 2101 Constitution Avenue, N.W., Washington, DC 20418.

m. Simmons, Marvin D., "Assessment of Geotechnical Factors Affecting the Stability of the Martins Fork Dam," May 1978. Available from U.S. Army Engineer District Nashville, P.O. Box 1070, Nashville, TN 37202.

n. Janbu, N., "Slope Stability Computations," Embankment Dam Engineering, Casagrande Volume, 1973, John Wiley and Sons, 605 Third Ave., New York, NY 10016.

o. Morgenstern, N.R. and Price, V.E., "The Analysis of the Stability of General Slip Surfaces," Geotechnique, Vol. No. 15, March 1965 Available from The Institute of Civil Engineers, Great George St., London, S.W. 1, England.

4. Action. For design and investigation of concrete structures, the assessment of sliding stability on rock and soil foundations should use the procedures outlined in the following paragraphs. The following guidance on sliding stability analyses has evolved from over two decades of experience in the design of substructures on foundations with weak sliding resistance.

5. Summary. This ETL prescribes guidance, developed from presently acceptable structural and geotechnical principles, in the form of equations for evaluating the factor of safety of single and multiple plane failure surfaces under both static and seismic loading conditions. Basic considerations for determining shear strength input parameters for the analysis are discussed. Minimum required factors of safety are established for both the static and seismic loading conditions. Background describing the development of the previously used shear-friction and resistance to sliding design criteria for evaluating the sliding stability of gravity hydraulic structures, and the basic reasons for replacing the old criteria, are included in inclosure one. Example problems for single and multiple wedge systems are presented in inclosure two. An alternate method of analysis is discussed in inclosure three.

6. Design Process.

a. Analysis. An adequate assessment of sliding stability must account for the basic structural behavior, the mechanism of transmitting compressive and shearing loads to the foundation, the reaction of the foundation to such loads, and the secondary effects of the foundation behavior on the structure.

b. Coordination. A fully coordinated team of geotechnical and structural engineers and geologists should insure that the result of the sliding analyses is properly integrated into the overall design of the substructure. Some of the critical aspects of the design process which require coordination are:

(1) Preliminary estimates of geotechnical data, subsurface conditions and types of substructures.

(2) Selection of loading conditions, loading effects, potential failure mechanisms and other related features of the analytical models.

(3) Evaluation of the technical and economic feasibility of alternative substructures.

(4) Refinement of the preliminary substructure configuration and proportions to reflect consistently the results of detailed geotechnical site explorations, laboratory testing and numerical analyses.

(5) Modification to the substructure configuration or features during construction due to unexpected variations in the foundation conditions.

7. Determining Foundation Strength Parameters.

a. General. The determination of foundation strength parameters is the most difficult geotechnical element of the assessment of sliding stability. This determination is made by analysis of the most appropriate laboratory and/or in-situ strength tests on representative foundation samples coupled with intimate knowledge of the geologic structure of a rock foundation or inhomogeneities of a soil foundation.

b. Field Investigation. The field investigation must be a continual process starting with the preliminary geologic review of known conditions, progressing to a detailed boring program and sample testing program and concluding at the end of construction with a safe and operational structure. The scope of investigation and sampling should be based on an assessment of inhomogeneity or geologic structural complexity. For example, the extent of the investigation could vary from quite limited (where the foundation material is strong even along the weakest potential failure planes) to quite extensive and detailed where weak zones or seams exist. However, it must be recognized that there is a certain minimum of investigation necessary to determine that weak zones are not present in the foundation. Undisturbed samples are required to determine the engineering properties of the foundation materials, demanding extreme care in application and sampling methods. Proper sampling is a combination of science and art, many procedures have been standardized but alteration and adaptation of techniques are often dictated by specific field procedures as discussed in EM 1110-1-1801, "Geological Investigations," EM 1110-2-1803, "Subsurface Investigations, Soils," and EM 1110-2-1907, "Soil Sampling."

c. Strength Testing. The nearly infinite number of combinations of soil and rock properties and rock structural conditions preclude a standardized universal approach to strength testing. Before any soil or rock testing is initiated, the geotechnical design engineer and the geologists responsible for formulating the testing program must clearly define the purpose of each test to themselves and to the persons who will supervise the testing. It is imperative to use all available data such as geological and geophysical studies when selecting representative samples for testing. Decisions must be made concerning the need for in-situ testing. Soil testing procedures are discussed in EM 1110-2-1906, "Laboratory Soils Testing." Rock testing procedures are discussed in the Rock Testing Handbook and in the International Society of Rock Mechanics, "Suggested Methods for Determining Shear Strength." These testing methods may be modified as appropriate to fit the circumstances of the project. (References 3j and 3l)

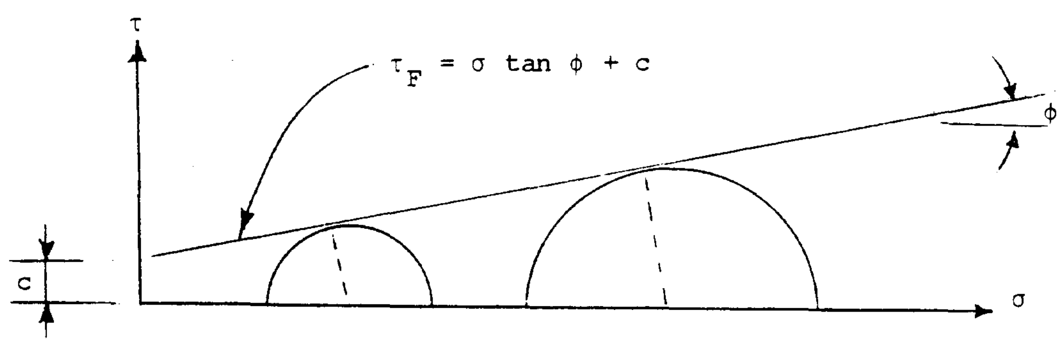
d. Design Shear Strengths. Shear strength values used in sliding analyses are determined from available laboratory and field tests, and judgment. Information in EM 1110-2-1902 "Stability of Earth and Rockfill Dams," on types of soils type tests and selection of design shear strengths should be used where appropriate. There is no equivalent Engineering Manual which provides information on appropriate types of rock tests and selection of shear strengths. It is important to select the types of tests based upon the probable mode of failure. Generally, strengths on rock discontinuities would be used with an active wedge and beneath the structure. A combination of strengths on discontinuities and/or intact rock strengths would be used with a passive wedge.

8. Method of Analysis.

a. Definition of Factor of Safety. The guidance in this ETL is based on modern principles of structural and geotechnical mechanics which apply a safety factor to the material strength parameters in a manner which places the forces acting on the structure and foundation wedges in sliding equilibrium. The factor of safety (FS) is defined as the ratio of the shear strength (τ_F) and the applied shear stress (τ) according to Equations one and two:

$$\frac{\tau_F}{\tau} = FS \qquad (1)$$

$$\frac{\tau_F}{FS} = \frac{\sigma \tan\phi}{FS} + \frac{c}{FS} \qquad (2)$$



Failure Envelope

b. Basic Concepts and Principles.

(1) A sliding mode of failure will occur along a presumed failure surface when the applied shearing force (T) exceeds the resisting shearing forces (T_F) The failure surface can be any combination of plane and curved surfaces, but for simplicity, all failure surfaces are assumed to be planes which form the

bases of wedges. The critical failure surface with the lowest safety factor, is determined by an iterative process.

(2) Sliding stability of most concrete structures can be adequately assessed by using a limit equilibrium approach. Designers must exercise sound judgment in performing these analyses. Assumptions and simplifications are listed below:

(a) A two-dimensional analysis is presented. These principles should be extended if unique three dimensional geometric features and loads critically affect the sliding stability of a specific structure.

(b) Only force equilibrium is satisfied in this analysis. Moment equilibrium is not used. The shearing force acting parallel to the interface of any two wedges is assumed to be negligible. Therefore the portion of the failure surface at the bottom of each wedge is only loaded by the forces directly above or below it. There is no interaction of vertical effects between the wedges. Refer to references 3n and 3o for a detailed discussion concerning the effects of moment equilibrium and shear forces acting at the interface.

(c) Analyses are based on assumed plane failure surfaces. The calculated safety factor will be realistic only if the assumed failure mechanism is kinematically possible.

(d) Considerations regarding displacements are excluded from the limit equilibrium approach. The relative rigidity of different foundation materials and the concrete substructure may influence the results of the sliding stability analysis. Such complex structure-foundation systems may require a more intensive sliding investigation than a limit equilibrium approach. The effects of strain compatibility along the assumed failure surface may be included by interpreting data from in-situ tests, laboratory tests and finite element analyses.

(e) A linear relationship is assumed between the resisting shearing force and the normal force acting along the failure surface beneath each wedge.

c. Analytical Techniques for Multi-wedge Systems.

(1) A derivation of the governing wedge equation for a typical wedge is shown on figures one through nine. The governing wedge equation is shown on figures six and seven.

(2) The following approach to evaluating sliding stability of concrete structures is based on the definition of safety factor and engineering principles discussed above. Examples of typical static loading conditions for single and multiple wedge systems are presented in inclosure two.

(3) A general procedure for analyzing multi-wedge systems includes:

(a) Assuming a potential failure surface which is based on the stratification, location and orientation, frequency and distribution of discontinuties of the foundation material, and the configuration of the substructure.

(b) Dividing the assumed slide mass into a number of wedges, including a single structural wedge.

(c) Drawing free body diagrams which show all the forces assumed to be acting on each wedge.

(d) Solving for the safety factor by either direct or iterative methods.

(4) The analysis proceeds by assuming trial values of the safety factor and unknown inclinations of the slip path so the governing equilibrium conditions, failure criterion and definition of safety factor are satisfied (see Figure 7). An analytical or a graphical procedure may be used for this iterative solution.

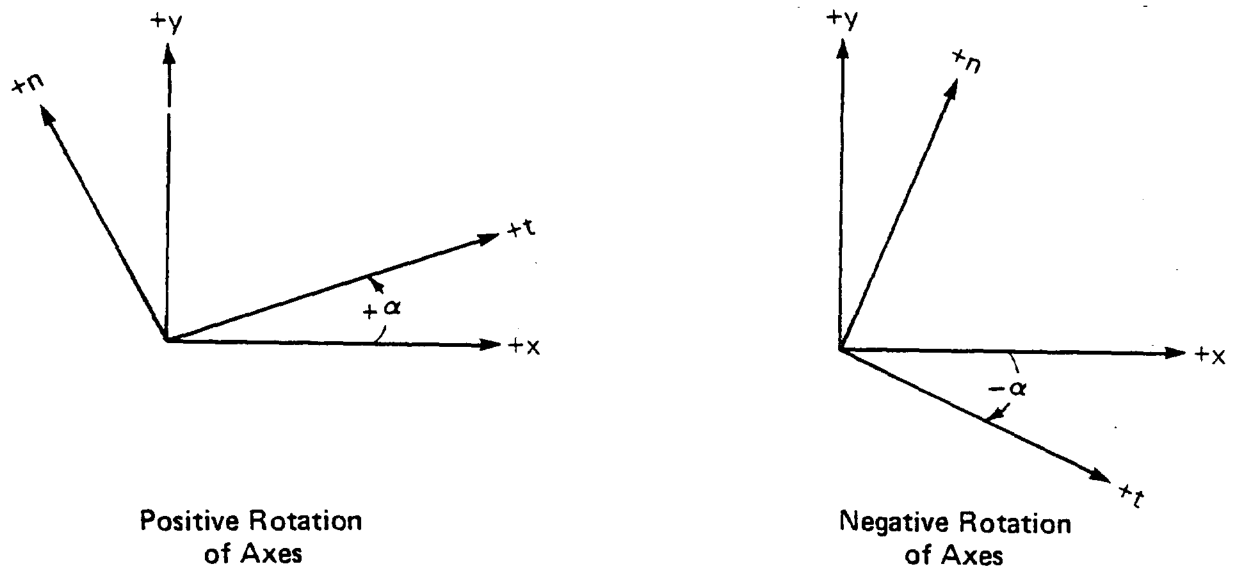
d. Design Considerations. Some special considerations for applying the general wedge equation to specific site conditions are discussed below.

(1) The interface between the group of active wedges and the structural wedge is assumed to be a vertical plane located at the heel of the structural wedge and extending to the base of the structural wedge. The magnitudes of the active forces depend on the actual values of the safety factor and the inclination angles (α) of the slip path. The inclination angles, corresponding to the maximum active forces for each potential failure surface, can be determined by independently analyzing the group of active wedges for a trial safety factor. In rock the inclination may be predetermined by discontinuities in the foundation. The general equation only applies directly to active wedges with assumed horizontal active forces.

(2) The governing wedge equation is based on the assumption that shearing forces do not act on the vertical wedge boundaries, hence there can only be one structural wedge because concrete structures transmit significant shearing forces across vertical internal planes. Discontinuities in the slip path beneath the structural wedge should be modeled by assuming an average slip-plane along the base of the structural wedge.

(3) The interface between the group of passive wedges and the structural wedge is assumed to be a vertical plane located at the toe of the structural wedge and extending to the base of the structural wedge. The magnitudes of the passive forces depend on the actual values of the safety factor and the inclination angles of the slip path. The inclination angles, corresponding to the minimum passive forces for each potential failure mechanism, can be determined by independently analyzing the group of passive wedges for a trial safety factor. The general equation only applies directly to passive wedges with assumed horizontal passive forces. When passive resistance is used special considerations must be made. Rock that may be subjected to high

Sliding Stability Analysis of
a General Wedge System



The equations for sliding stability analysis of a general wedge system are based on the right hand sign convention which is commonly used in engineering mechanics. The origin of the coordinate system for each wedge is located in the lower left hand corner of the wedge. The x and y axes are horizontal and vertical respectively. Axes which are tangent (t) and normal (n) to the failure plane are oriented at an angle (α) with respect to the +x and +y axes. A positive value of α is a counter-clockwise rotation, a negative value of α is a clockwise rotation.

Figure 1. Sign Convention for Geometry

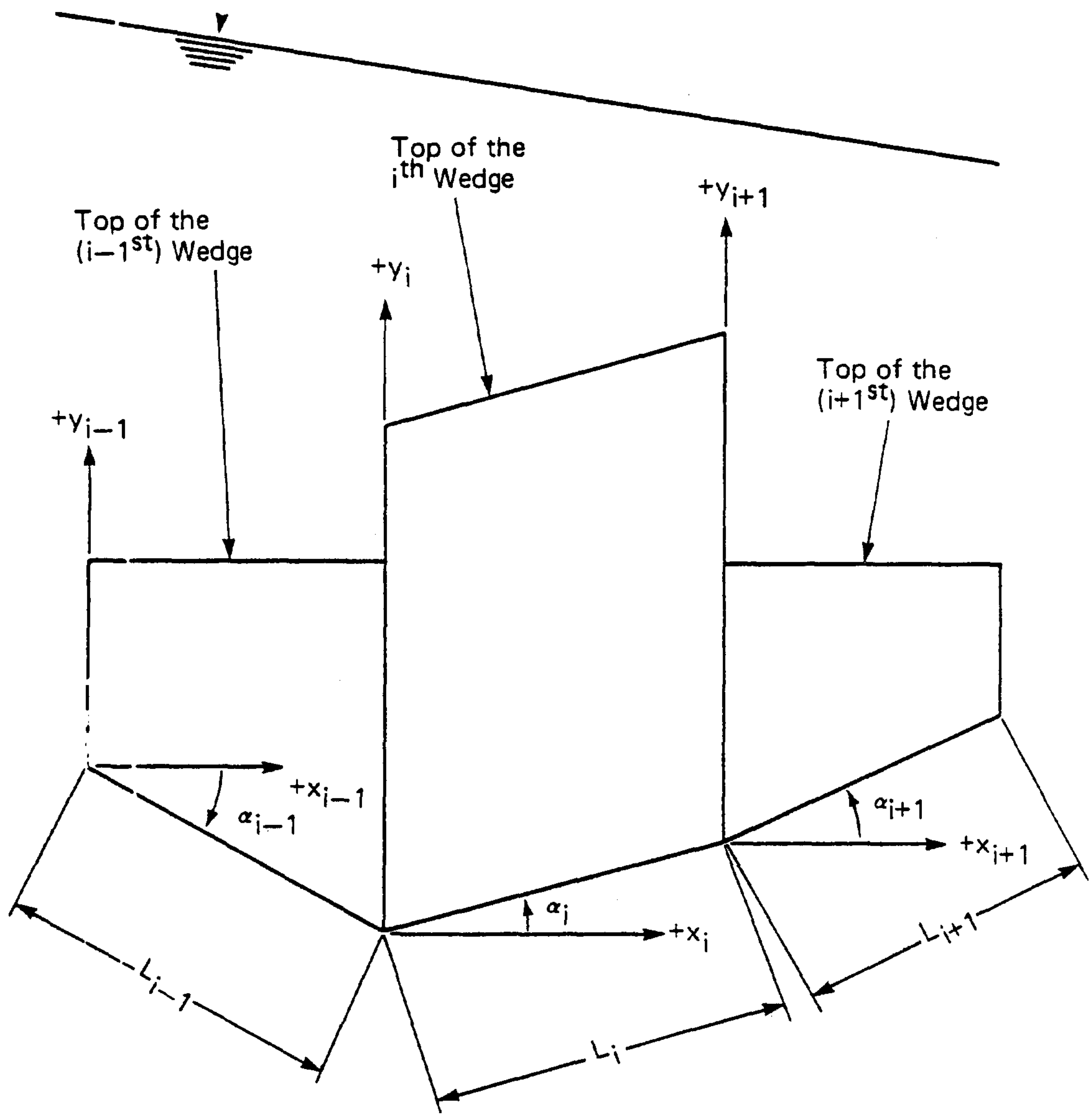


Figure 2. Geometry of the Typical ith Wedge and Adjacent Wedges

Sliding Stability Analysis of
a General Wedge System

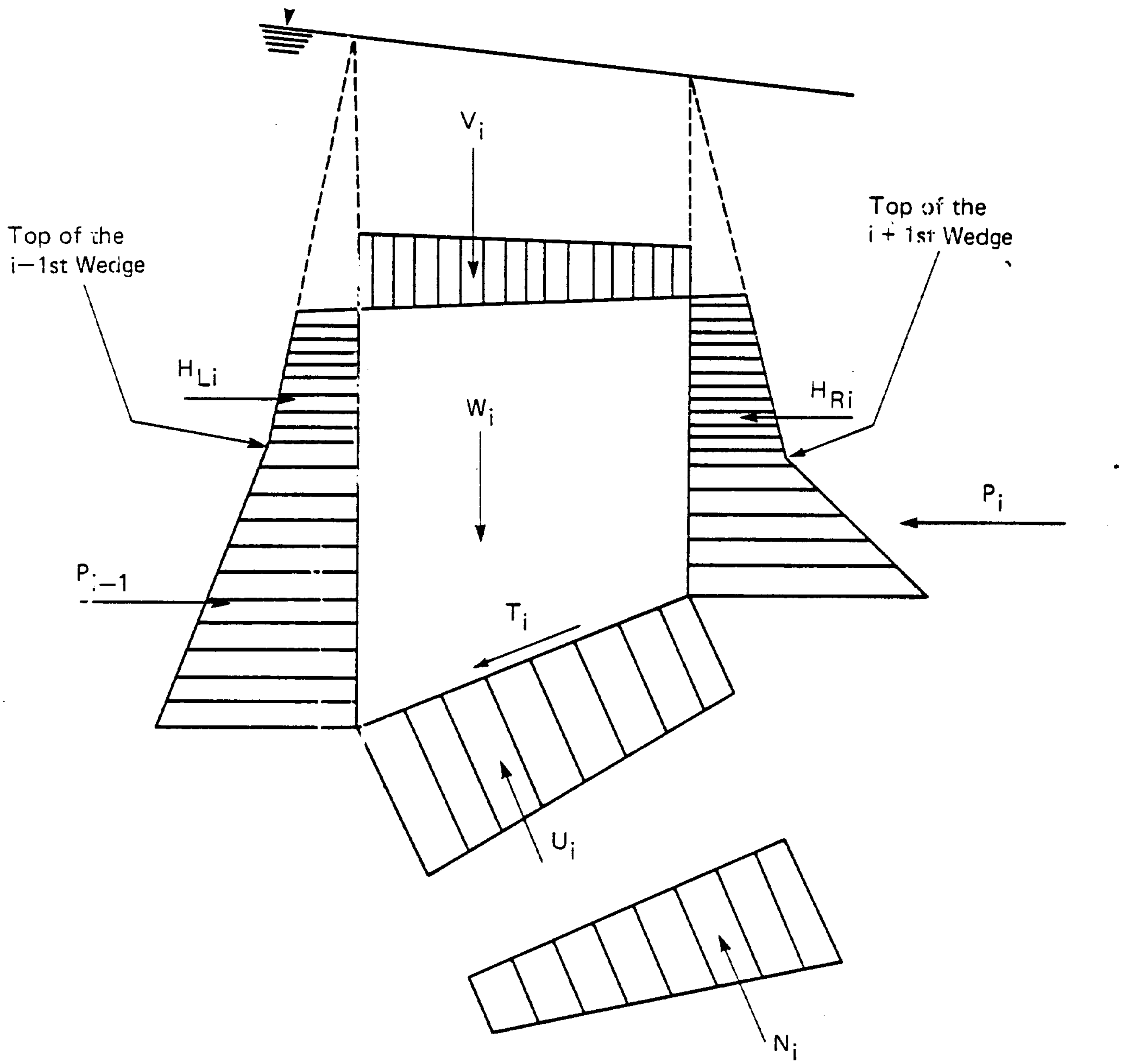


Figure 3. Distribution of Pressures and Resultant Forces Acting on a Typical Wedge

Sliding Stability Analysis of
a General Wedge System

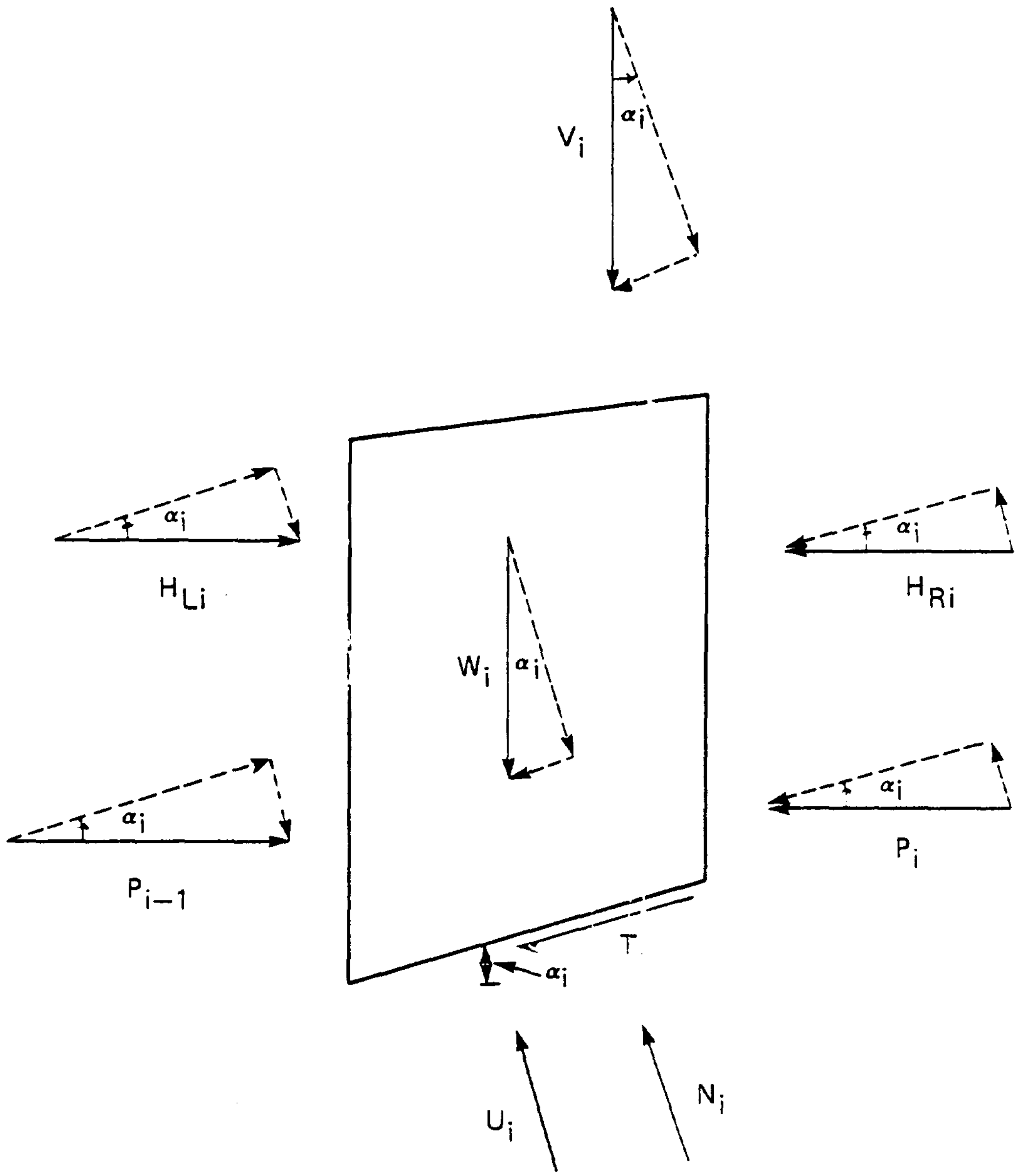
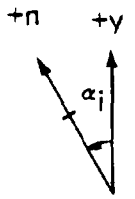


Figure 4. Free Body Diagram of the i^{th} Wedge

Sliding Stability Analysis of a General Wedge System

Equilibrium Equations

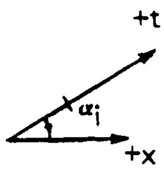


$$\Sigma F_n = 0$$

$$0 = N_i + U_i - W_i \cos\alpha_i - V_i \cos\alpha_i - H_{Li} \sin\alpha_i + H_{Ri} \sin\alpha_i + \ldots$$

$$\ldots - P_{i-1} \sin\alpha_i + P_i \sin\alpha_i$$

$$N_i = (W_i + V_i)\cos\alpha_i - U_i + (H_{Li} - H_{Ri})\sin\alpha_i + (P_{i-1} - P_i)\sin\alpha_i \qquad (3)$$



$$\Sigma F_t = 0$$

$$0 = -T_i - W_i \sin\alpha_i - V_i \sin\alpha_i + H_{Li}\cos\alpha_i - H_{Ri}\cos\alpha_i + \ldots$$

$$\ldots + P_{i-1}\cos\alpha_i - P_i \cos\alpha_i$$

$$T_i = (H_{Li} - H_{Ri})\cos\alpha_i - (W_i + V_i)\sin\alpha_i + (P_{i-1} - P_i)\cos\alpha_i \qquad (4)$$

Mohr-Coulomb Failure Criterion

$$T_F = N_i \tan\phi_i + c_i L_i \qquad (5)$$

Safety Factor Definition

$$FS_i = \frac{T_F}{T_i} = \frac{N_i \tan\phi_i + c_i L_i}{T_i} \qquad (6)$$

Figure 5. Derivation of the General Equation

Sliding Stability Analysis of
a General Wedge System

Governing Wedge Equation

$$FS_i = \frac{\left\{(W_i + V_i)\cos\alpha_i - U_i + [(H_{Li} - H_{Ri}) + (P_{i-1} - P_i)]\sin\alpha_i\right\}\tan\phi_i + c_i L_i}{[(H_{Li} - H_{Ri}) + (P_{i-1} - P_i)]\cos\alpha_i - (W_i + V_i)\sin\alpha_i}$$

$$(P_{i-1} - P_i)\left(\cos\alpha_i - \sin\alpha_i \frac{\tan\phi_i}{FS_i}\right) = [(W_i + V_i)\cos\alpha_i - U_i + (H_{Li} - H_{Ri})\sin\alpha_i]\frac{\tan\phi_i}{FS_i} + \ldots$$

$$\ldots + \frac{c_i}{FS_i}L_i - (H_{Li} - H_{Ri})\cos\alpha_i + (W_i + V_i)\sin\alpha_i$$

$$(P_{i-1} - P_i) = \frac{[(W_i + V_i)\cos\alpha_i - U_i + (H_{Li} - H_{Ri})\sin\alpha_i]\frac{\tan\phi_i}{FS_i} - (H_{Li} - H_{Ri})\cos\alpha_i + (W_i + V_i)\sin\alpha_i + \frac{c_i}{FS_i}L_i}{\left(\cos\alpha_i - \sin\alpha_i\frac{\tan\phi_i}{FS_i}\right)} \quad (7)$$

NOTE: A negative value of the difference $(P_{i-1} - P_i)$ indicates that the applied forces acting on the i^{th} wedge exceed the forces resisting sliding along the base of the wedge. A positive value of the difference $(P_{i-1} - P_i)$ indicates that the applied forces acting on the i^{th} wedge are less than the forces resisting sliding along the base of that wedge.

Figure 6. Derivation of the General Equation

Sliding Stability Analysis for a General Wedge System

Solution for the Safety Factor

The governing equation for $(P_{i-1} - P_i)$ applies to the *individual* wedges

$$(P_{i-1} - P_i) = \frac{[(W_i + V_i)\cos\alpha_i - U_i + (H_{Li} - H_{Ri})\sin\alpha_i]\dfrac{\tan\phi_i}{FS_i} - (H_{Li} - H_{Ri})\cos\alpha_i + (W_i + V_i)\sin\alpha_i + \dfrac{c_i}{FS_i}L_i}{\left(\cos\alpha_i - \sin\alpha_i\dfrac{\tan\phi_i}{FS_i}\right)}$$

For the *system* of *wedges* to act as an integral failure mechanism, the *safety factors* for all wedges must be *identical*

$FS_1 = FS_2 = \ldots = FS_{i-1} = FS_i = FS_{i+1} = \ldots FS_N$

N = Number of wedges in the failure mechanism

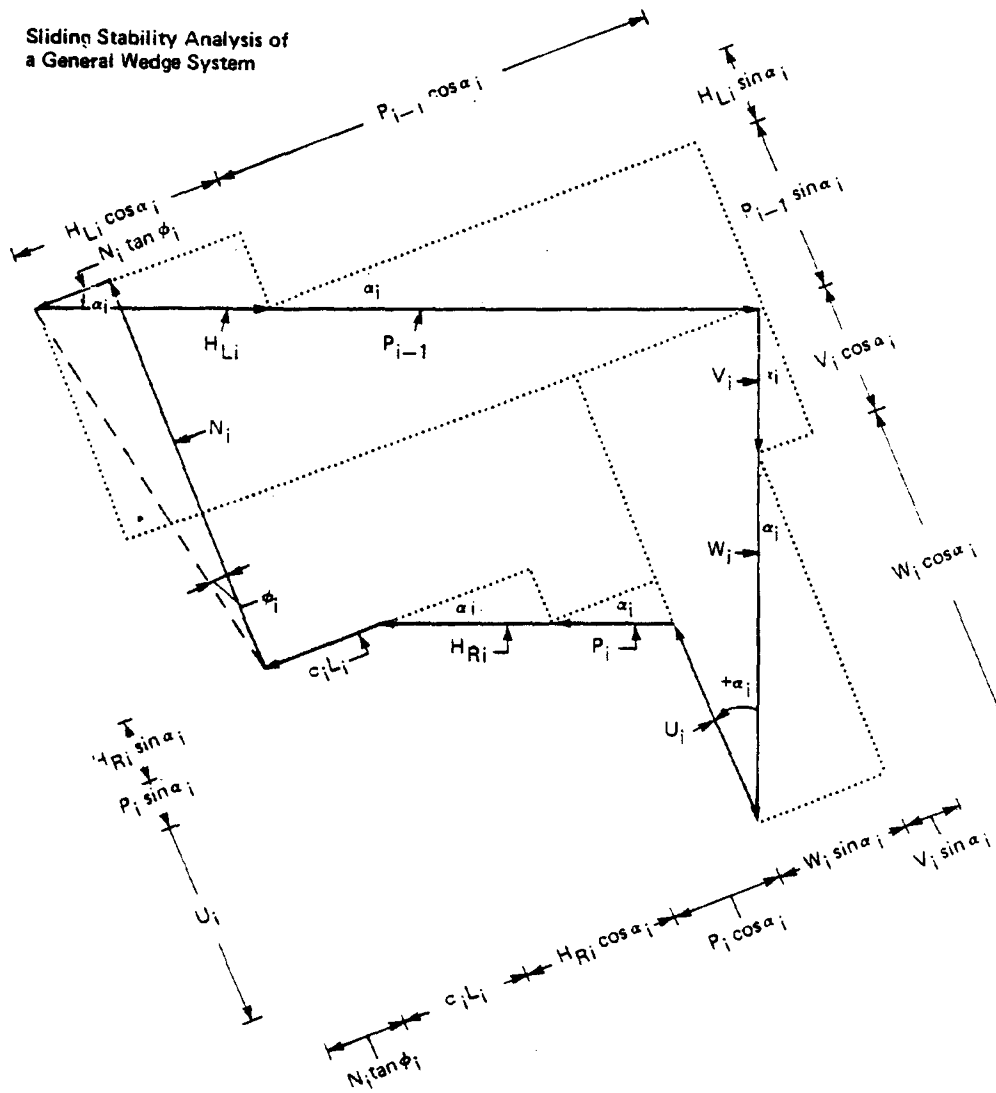
The *actual* safety factor (FS) for sliding equilibrium is determined by *satisfying overall horizontal equilibrium* $(\Sigma \bar{F}_H = 0)$ for the *entire* system of wedges

$$\sum_{i=1}^{N} (P_{i-1} - P_i) = 0$$

And: $P_o \equiv 0 \quad P_N \equiv 0$

Usually an *iterative solution* process is used to determine the actual safety factor for sliding equilibrium.

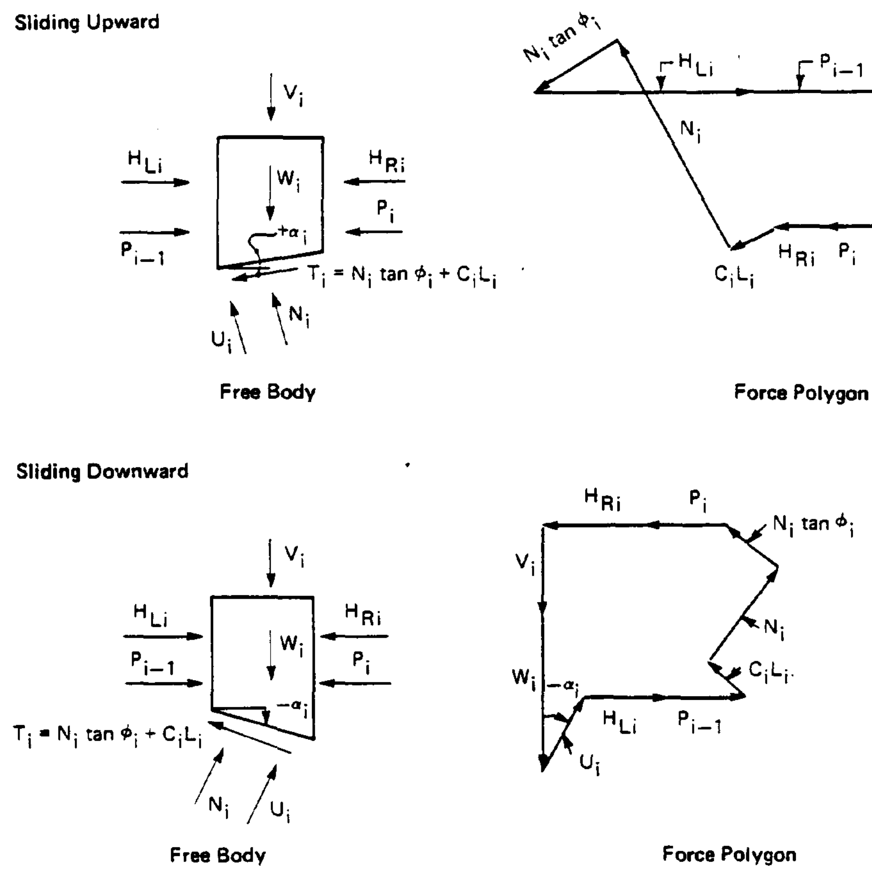
Figure 7. Derivation of the General Equation



A sliding stability analysis using the general wedge equation should yield results comparable to those obtained from graphical solutions using force polygons. This is clarified by the following discussion of the force polygon shown above

Figure 8. Force Polygon for a Typical Wedge

The angle (α_i) between the W_i and U_i vectors is a positive value for wedges sliding upward, and is a negative value for wedges sliding downward.



Refer to notes on following page.

Figure 9. Force Polygons for Upward and Downward Sliding.

NOTES FOR FIGURES 8 and 9

1. The relationships from the typical force polygon are consistent with the analytical relationships previously developed for the Governing Wedge Equation.

2. The lines of dimensions on the force polygon lying parallel to the U_i and N_i vectors represent the summation of forces normal to the failure plane (identical to equation three).

$$N_i = H_{Li} \sin \alpha_i + P_{i-1} \sin \alpha_i + V_i \cos \alpha_i + W_i \cos \alpha_i - U_i - P_i \sin \alpha_i - H_{Ri} \sin \alpha_i$$

3. The lines of dimensions on the force polygon shown on Figure Nine lying parallel to the $C_i L_i$ and $N_i \tan \phi_i$ vectors represent the summation of forces parallel to the failure plane (identical to equation four).

$$P_{i-1} \cos \alpha_i = V_i \sin \alpha_i + W_i \sin \alpha_i + P_i \cos \alpha_i + H_{Ri} \cos \alpha_i + C_i L_i + N_i \tan\phi_i - H_{Li} \cos \alpha_i$$

4. These two equations can be combined with the safety factor definition for a typical wedge to obtain the Governing Wedge Equation.

$$P_{i-1} - P_i = \frac{\{(W_i+V_i)\cos\alpha_i - U_i + (H_{Li}-H_{Ri})\sin\alpha_i\}\frac{\tan\phi_i}{FS_i} - (H_{Li}-H_{Ri})\cos\alpha_i + (W_i+V_i)\sin\alpha_i + \frac{ci}{FS_i} L_i}{\left(\cos\alpha_i - \sin\alpha_i \frac{\tan\phi_i}{FS_i}\right)}$$

velocity water scouring should not be used unless amply protected. Also, the compressive strength of the rock layers must be sufficient to develop the wedge resistance. In some cases wedge resistance should not be assumed without resorting to special treatment such as installing rock anchors.

(4) Sliding analyses should consider the effects of cracks on the active side of the structural wedge in the foundation material due to differential settlement, shrinkage or joints in a rock mass. The depth of cracking in cohesive foundation material can be estimated in accordance with equations eight through ten:

$$d_c = \frac{2c_d}{\gamma} \tan\left(45 + \frac{\phi_d}{2}\right) \quad (8)$$

$$c_d = \frac{c}{FS} \quad (9)$$

$$\phi_d = \tan^{-1}\left(\frac{\tan\phi}{FS}\right) \quad (10)$$

The value (d_c) in a cohesive foundation cannot exceed the embedment of the structural wedge. The depth of cracking in massive strong rock foundations should be assumed to extend to the base of the structural wedge. Shearing resistance along the crack should be ignored and full hydrostatic pressure should be assumed to act at the bottom of the crack. The hydraulic gradient across the base of the structural wedge should reflect the presence of a crack at the heel of the structural wedge.

(5) The effects of seepage forces should be included in the sliding analysis. Analyses should be based on conservative estimates of uplift pressures. Estimates of uplift pressures on the wedges can be based on the following assumptions:

(a) The uplift pressure acts over the entire area of the base.

(b) If seepage from headwater to tailwater can occur across a structure, the pressure head at any point should reflect the head loss due to water flowing through a medium. The approximate pressure head at any point can be determined by the line-of-seepage method. This method assumes that the head loss is directly proportional to the length of the seepage path. The seepage path for the structural wedge extends from the upper surface (or internal groundwater level) of the uncracked material adjacent to the heel of the structure, along the embedded perimeter of the structural wedge, to the upper surface (or internal groundwater level) adjacent to the toe of the structure. Referring to figure ten, the seepage distance is defined by points a, b, c, and d. The pressure head at any point is equal to the initial total head minus the product of the hydraulic gradient times the seepage path distance to the point in question, minus the elevation head. The pressure head is defined as the height to which water rises in a piezometer located at the point under consideration. The initial total head is the head differential between headwater and tailwater. The elevation head is the vertical distance between the point being considered and the tailwater elevation (negative if below tailwater or positive if above). Estimates of pressure heads for the active and passive wedges should be consistent with those of the heel and toe of the structural wedge. For a more detailed discussion of the line-of-seepage method, refer to EM 1110-2-2501, Floodwalls. For the majority of structural stability computations, the line-of-seepage is considered sufficiently accurate. However, there may be special situations where the flow net method is required to evaluate seepage problems.

(c) Uplift pressures on the base of the structural wedge can be reduced by foundation drains. The pressure heads beneath the structural wedge developed from the line-of-seepage analysis should be modified to reflect the effects of the foundation drains. A maximum reduction in pressure head along the line of foundation drains equal to the pressure head at the structure toe plus 25-50 percent of the difference between the undrained pressure head at the toe and that at the line of drains may be assumed. The uplift pressure across the base of the structural wedge usually varies from the undrained pressure head at the heel to the assumed reduced pressure head at the line of drains to the undrained pressure head at the toe, as shown in figure ten. Uplift forces used for the sliding analyses should be selected in consideration of conditions which are presented in the applicable design memoranda. For a more detailed discussion of uplift under gravity dams, refer to EM 1110-2-2200, Gravity Dams.

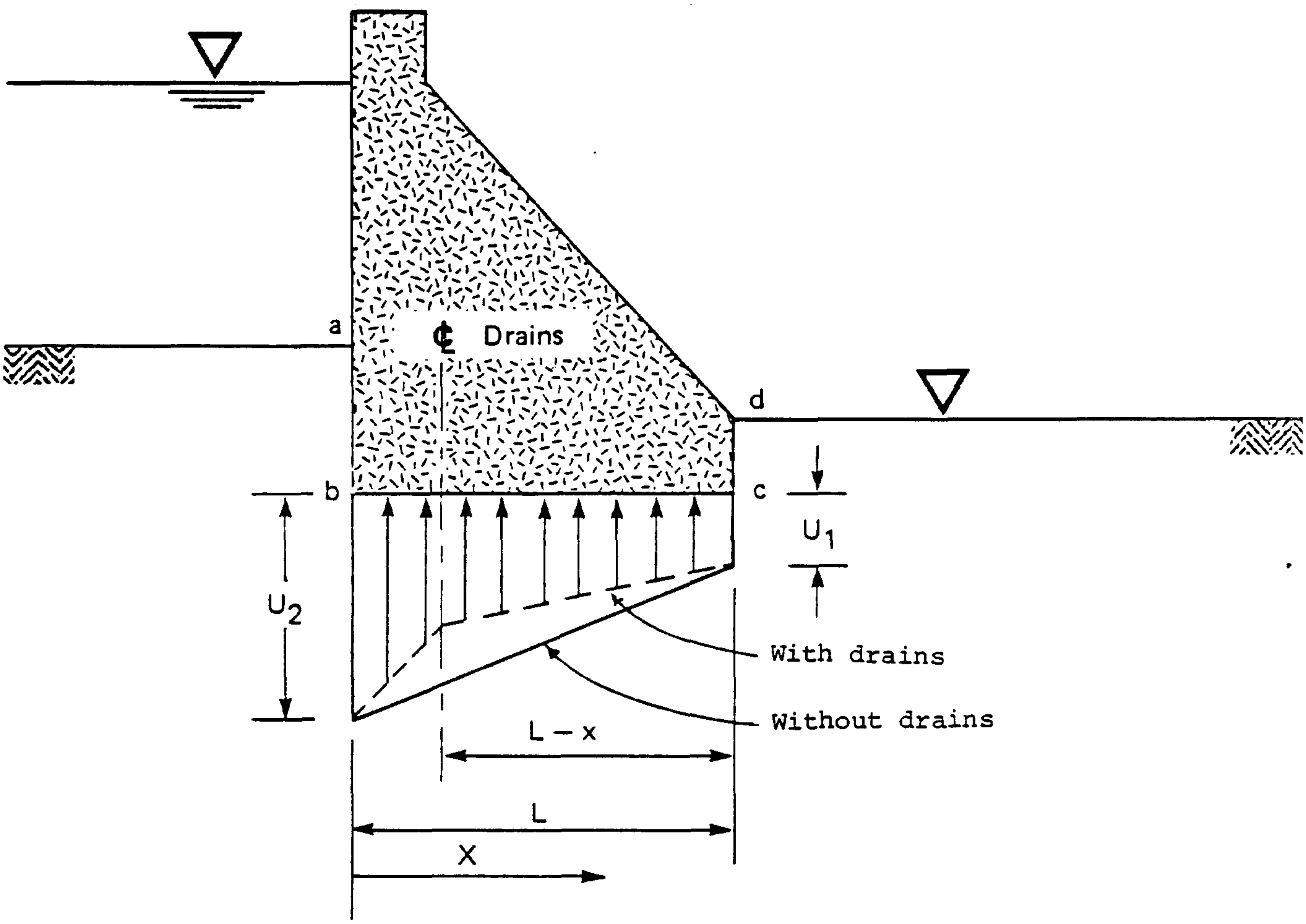
(6) As stated previously, requirements for rotational equilibrium are not directly included in the general wedge equation. For some load cases, the vertical component of the resultant applied loads will lie outside the kern of the base area, and a portion of the structural wedge will not be in contact with the foundation material. The sliding analysis should be modified for these load cases to reflect the following secondary effects due to <u>coupling of sliding and overturning</u> behavior.

(a) The uplift pressure on the portion of the base which is not in contact with the foundation material should be a uniform value which is equal to the maximum value of the hydraulic pressure across the base, (except for instantaneous load cases such as due to seismic forces).

(b) The cohesive component of the sliding resistance should only include the portion of the base area which is in contact with the foundation material.

e. <u>Seismic Sliding Stability</u>.

(1) The sliding stability of a structure for an earthquake-induced base motion should be checked by assuming the specified horizontal earthquake



Pressure Head at Drains = $U_x = U_1 + R \left(\frac{L-x}{L}\right) (U_2 - U_1)$

U_1 = Pressure Head at Toe

U_2 = Pressure Head at Heel

R = Constant {100 - (25% → 50%)}

Figure 10. Uplift Pressures

acceleration, and the vertical earthquake acceleration if included in the analysis, to act in the most unfavorable direction (figure 11). The earthquake-induced forces on the structure and foundation wedges may then be determined by a rigid body analysis.

(2) For the rigid body analysis the horizontal and vertical forces on the structure and foundation wedges may be determined by using the following equations:

$$\Sigma H = M\ddot{x} + m\ddot{x} + Hs \qquad (11)$$

$$\Sigma V = Mg - m\ddot{y} - U \qquad (12)$$

M = mass of structure and wedges, weight/g
m = added mass of reservoir and/or adjacent soil
g = acceleration of gravity
$\ddot{x}$ = horizontal earthquake acceleration
$\ddot{y}$ = vertical earthquake acceleration
Hs = resultant horizontal static forces
U = hydrostatic uplift force

(3) The horizontal earthquake acceleration can be obtained from seismic zone maps (ER 1110-2-1806 "Earthquake Design and Analysis for Corps of Engineers Dams") or, in the case where a design earthquake has been specified for the structure, an acceleration developed from analysis of the design earthquake. Guidance is being prepared for the latter type of analysis and will be issued in the near future; until then, the seismic coefficient method is the most expedient method to use. The vertical earthquake acceleration is normally neglected but can be taken as two-thirds of the horizontal acceleration if included in the analysis.

(4) The added mass of the reservoir and soil can be approximated by Westergaard's parabola (EM 1110-2-2200 "Gravity Dam Design") and the Mononobe-Okabe method (EM 1110-2-2502 "Retaining Walls"), respectively. The structure should be designed for a simultaneous increase in force on one side and decrease on the opposite side of the structure when such can occur.

9. Required Factors of Safety.

a. Factors of Safety. For major concrete structures (dams, lockwalls, basin walls which retain a dam embankment, etc.) the minimum required factor of safety for normal static loading conditions is 2.0. The minimum required factor of safety for seismic loading conditions is 1.3. Flood walls and retaining walls are excepted from the provisions of this paragraph; refer to EM 1110-2-2501 and EM 1110-2-2502 for a discussion of safety factors for those structures. Any relaxation of these values will be accomplished only with the approval of DAEN-CWE and should be justified by comprehensive foundation studies of such nature as to reduce uncertainties to a minimum.

b. Past Practice. Prior to issuing this ETL, the minimum required factor of safety for static loading conditions (as calculated by the shear friction method) was four. The primary reasons for use of this conservative factor of safety were the uncertainty in determining rock shear strength parameters and the peak shear strengths from tests on intact rock. The minimum required factor of safety for static loading conditions has been reduced to two for the reasons discussed in inclosure one and the following:

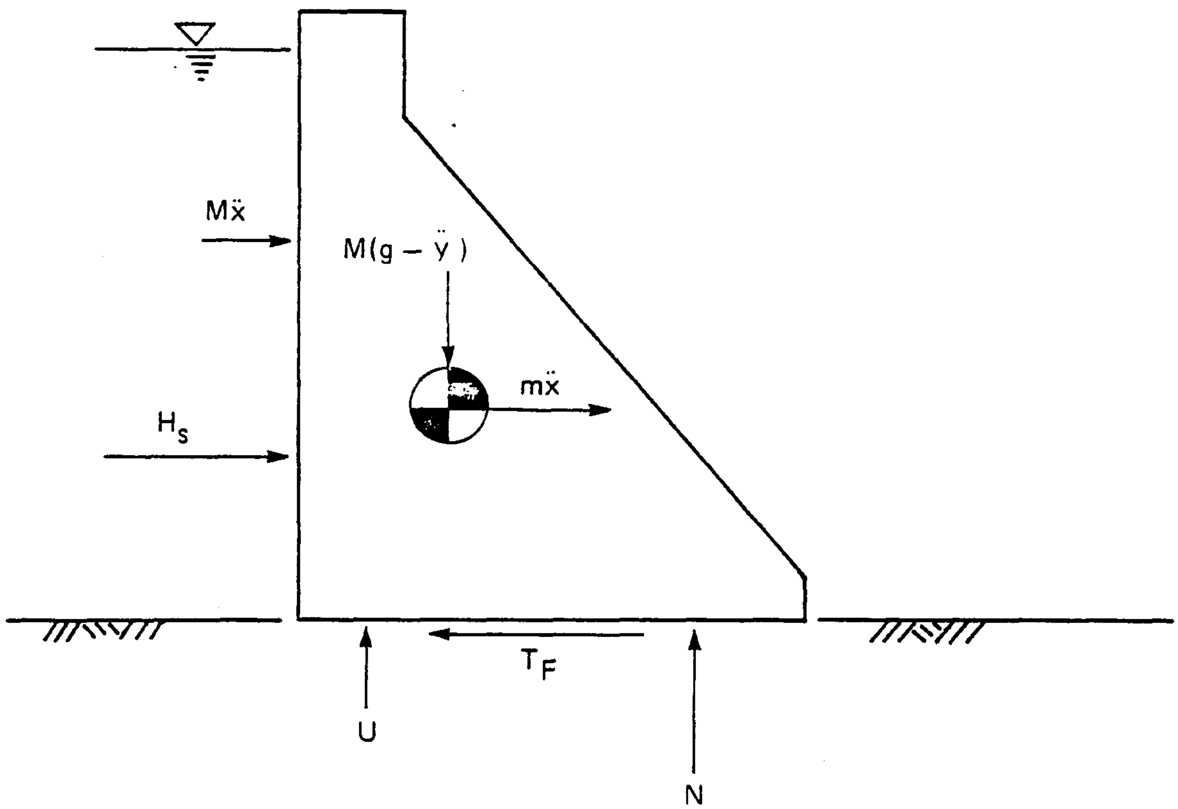


Figure 11. Seismically Loaded Gravity Dam

LIST OF SYMBOLS

Symbol	Definition
F	Forces.
H	In general, any horizontal force applied above the top or below the bottom of the adjacent wedge.
L	Length of wedge along the failure surface.
N	The resultant normal force along the failure surface.
P	The resultant pressure acting on a vertical face of a typical wedge.
FS	The factor-of-safety.
T	The shearing force acting along the failure surface.
T_F	The maximum resisting shearing force which can act along the failure surface.
U	The uplift force exerted along the failure surface of the wedge.
V	Any vertical force applied above the top of the wedge.
W	The total weight of water, soil or concrete in the wedge.
c	Cohesion.
α	The angle between the inclined plane of the potential failure surface and the horizontal (positive counter-clockwise).
ϕ	The angle of shearing resistance, or internal friction.
γ	Weight per unit volume.

LIST OF SYMBOLS

Symbol	Definition
σ	Normal stress.
τ	Shear stress.
τ_F	Shear strength.

NOTE: Subscripts containing(i, i-1, i, i+1, -----) refer to body forces, surface forces or dimensions associated with the i^{th} wedge.

Subscripts containing Ri or Li refer to the right or left side of the i^{th} wedge.

(1) Methods of sampling and sample testing have substantially improved and much better definition of soil and rock mass strengths are now possible. Of the above reasons, the capability of better definition of mass strength is, by far, the most important. Sampling techniques of two or three decades ago favored the collection of intact samples with little attention being given to core loss zones. Tests were usually performed on intact specimens and gave large values for cohesion and angles of internal friction. Testing of strengths along discontinuities such as bedding planes, joint planes and tests on joint filling materials were rarely performed. Tests were rarely carried beyond peak strength to determine ultimate and residual strengths. Current exploration practice is to emphasize obtaining samples from the weak zones. Tests are run on discontinuities and weak zones. Peak, ultimate and residual strengths are obtained. If necessary, in-situ tests are performed.

(2) Factors of safety less than four have been used for the design of the Waco, Proctor, Aliceville and Martins Fork projects (projects in Southwest Division, South Atlantic Division and Ohio River Division). Details concerning the design of the Martins Fork Project are available in reference 3m.

(3) In past and current stability analyses the three dimensional (side) effects exist, and are not accounted for; which results in additional safety.

c. General. Appropriate values of computed safety factors depend on the; (1) design condition being analyzed; (2) degree of confidence in design shear strength values; (3) consequence of failure; (4) thoroughness of investigation; (5) nature of structure-foundation interaction; (6) environmental conditions and quality of workmanship during construction; and (7) judgment based on past experience with similar structures. For example, for flood control structures the most critical loading condition usually is caused by a high reservoir level of infrequent occurrence, and for low-head navigation dams, the most critical loading condition with the greatest head differential is the normal operating condition, which exists most of the time.

FOR THE COMMANDER:

LLOYD A. DUSCHA, P. E.
Chief, Engineering Division
Directorate of Civil Works

3 Incl
1. Background
2. Examples
3. Alternate Method of Analysis

BACKGROUND

1. Previous Methods. Two of the approaches to the sliding stability analysis that have been used by the Corps of Engineers (CE), are the sliding resistance and shear-friction methods.

a. Sliding Resistance Method. The sliding resistance method is seldom used by the CE for current designs. This concept was the common criterion for evaluating sliding stability of gravity dams from approximately 1900 to the mid-1930's. Experience of the early dam designers had shown that the shearing resistance of very competent foundation material need not be investigated if the ratio of horizontal forces to vertical forces ($\Sigma H/\Sigma V$) is such that a reasonable safety factor against sliding results. The maximum ratio of $\Sigma H/\Sigma V$ is set at 0.65 for static loading conditions and 0.85 for seismic conditions.

b. Shear-Friction.

(1) The shear-friction method of analysis is the guidance currently used throughout the CE for evaluating sliding stability of gravity dams and mass concrete hydraulic structures. This method was introduced by Henny in 1933 (Reference 3k "Stability of Straight Concrete Gravity Dams"). The basic formula is

$$Q = \frac{S}{P} \qquad (1)$$

The shear-friction method was extended in later guidance.

The total resisting shear strength, S, was defined by the Coulomb equation:

$$S = S_1 + k\ (W-U) \qquad (2)$$

It is important to note that Henny considered only single, horizontal failure planes.

(2) Henny established the minimum shear-friction factor as four (4). Although the rationale for selecting this value is vague, it does appear to be the approximate average value of Q in Table eight of Reference 3k which compares the dimensions of an ideal dam, uplift forces, shear-friction safety factors, and nominal sliding factors.

(3) Records cannot be located to indicate adaptation of Henny's work into the Corps of Engineers sliding stability criteria. Nevertheless, the initial concept of defining the shear-friction factor as the ratio of the total resisting shear force acting along a horizontal failure plane to the maximum horizontal driving force can be attributed to Henny and thus technology of the 1930's.

(4) The earliest form of the shear-friction in official CE guidance is:

$$S_{s-f} = \frac{f\Sigma V + rSA}{\Sigma H}$$

This equation included a factor (r) by which S_1 was multiplied. This factor represents the ratio between average and maximum shear stresses. It was generally assumed to equal 0.5. This was a partial attempt to allow for possible progressive failure.

(5) The definition of the shear-friction factor was expanded to include the effect of inclined failure planes and embedment to resistance. The shear-friction factor, in the expanded form, was defined as:

$$S_{s-f} = \frac{R + P_p}{H} \quad (3)$$

Equations for R and P_p were derived for static equilibrium conditions that treated the downstream wedge and structure (including any foundation material beneath the structure but above the critical path) as being separate sliding bodies. The minimum acceptable shear-friction factor (S_{s-f}) required for CE design was specified as four (4).

2. Problems with Previous Design Criteria

a. Sliding Resistance. Limitations of the sliding resistance approach are:

(1) The criterion is valid only for structures with critical sliding failure along a horizontal plane.

(2) The limiting ratio of $\frac{\Sigma H}{\Sigma V} \leqq 0.65$ was only intended for structures founded on very competent rock.

b. Shear-Friction. Limitations of the shear-friction approach are:

(1) The shear-friction factor is defined as the ratio of the maximum horizontal base resistance plus a passive resistance that is composed of shear strength and weight components, to the horizontal force actually applied. The safety factor relative to sliding stability should be applied to the shear strength of the material rather than partially strength and partially weight components.

(2) The shear-friction factor for upslope sliding approaches infinity when the angle of inclination of the failure plane is equal to an angle of $(90 - \phi)$.

(3) The value of passive resistance (P_p) used in Equation three was defined as the maximum force which can be developed by the wedge acting independently from the forces acting on the structure. The structure and the passive wedge act as a compatible system which is in static equilibrium.

LIST OF SYMBOLS FOR INCLOSURE 1

Symbol	Definition
A	The portion of the critical potential failure surface which is in compression.
H	The summation of horizontal service loads to be applied to the structure.
k	The factor of shearing strength increase.
P	The water pressure on the projected area of the structure assumed to move and acting on a vertical plane normal to the direction of motion.
P_p	The passive resistance of the rock wedge at downstream toe.
Q	Factor of safety of shear.
R	The maximum horizontal driving force which can be resisted by the critical path.
r	The ratio between average and maximum shear stress.
S	Total resisting shear strength acting over the failure plane.
S_1	The total shear strength under conditions of no load.
S_{s-f}	The shear-friction factor of safety.
V	The summation of vertical service loads to be applied to the structure.
U	The uplift force under the sliding plane.
W	The weight of the structure above an assumed sliding plane.

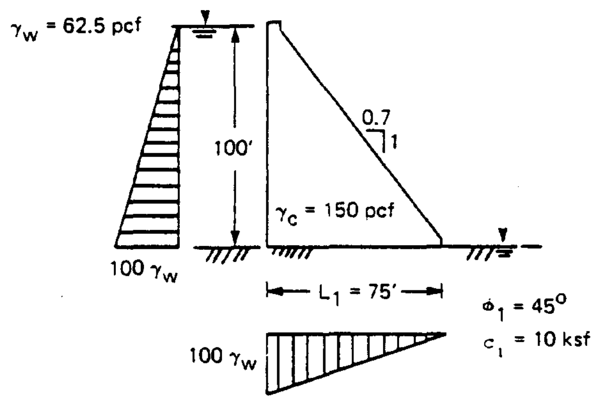
EXAMPLES

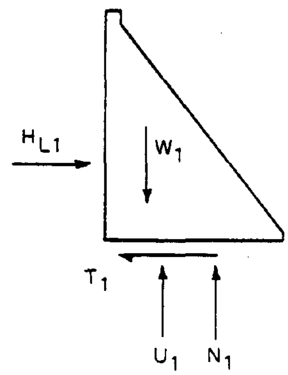
1. Examples of typical static loading conditions for single and multiple wedge systems are presented in this Inclosure.

2. These examples are provided to clearly demonstrate the procedure for applying the general wedge equation to the sliding analysis of single and multiple wedge systems. The variation of uplift pressure, orientation of failure planes, etc., used in the examples were only selected to simplify the calculations, and are not intended to represent the only conditions to be considered during the design of a hydraulic structure.

Example 1: Single Wedge

81 Determine the factor of safety against sliding for the following single wedge system.





Free Body Diagram

General Wedge Equation

$$P_{i-1} - P_i = \frac{[(W_i + V_i)\cos\alpha_i - U_i + (H_{Li} - H_{Ri})\sin\alpha_i]\dfrac{\tan\phi_i}{FS_i} - (H_{Li} - H_{Ri})\cos\alpha_i + (W_i + V_i)\sin\alpha_i + \dfrac{c_i}{FS_i}L_i}{\left(\cos\alpha_i - \sin\alpha_i\dfrac{\tan\phi_i}{FS_i}\right)}$$

Solve for Safety Factor (FS)

$i = 1 \quad H_{R1} = 0 \quad V_1 = 0 \quad P_o \quad P_1 = 0 \quad \alpha_1 = 0 \quad \cos\alpha_1 = 1 \quad \sin\alpha_1 = 0$

$$o = (W_1 - U_1)\frac{\tan 45}{FS} - H_{L1} + \frac{c_1}{FS}L_1$$

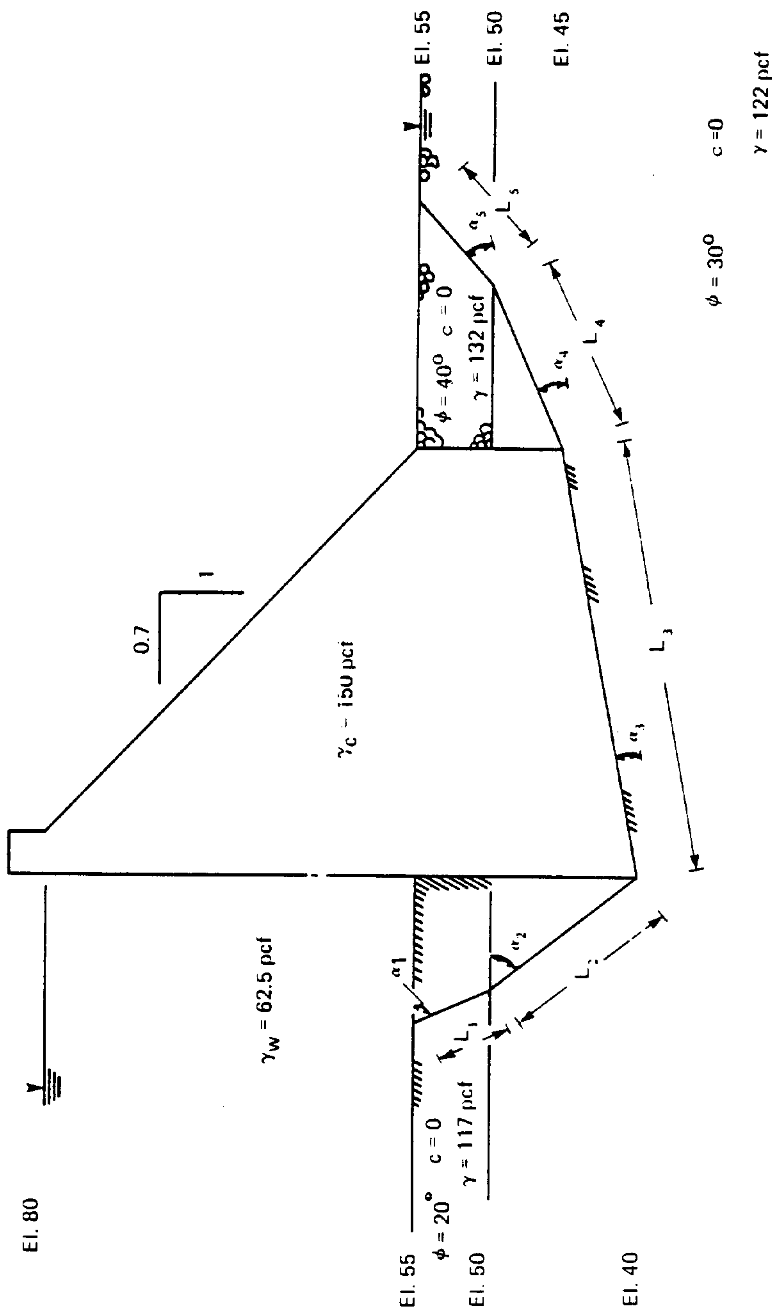
$H_{L1} = \frac{1}{2}(100)^2\,\gamma_w = 312.5^k \qquad U_1 = \frac{1}{2}(75)(100)\,\gamma_w = 234.4^k\uparrow \qquad W_1 = 603.8^k\downarrow$

$$FS = \frac{(W_1 - U_1)\tan 45^\circ + c_1 L_1}{H_{L1}}$$

$$FS = \frac{(603.8 - 234.4)(1) + 10(75)}{312.5} = \frac{(369.4 + 750)}{312.5} = 3.58$$

FS = 3.58

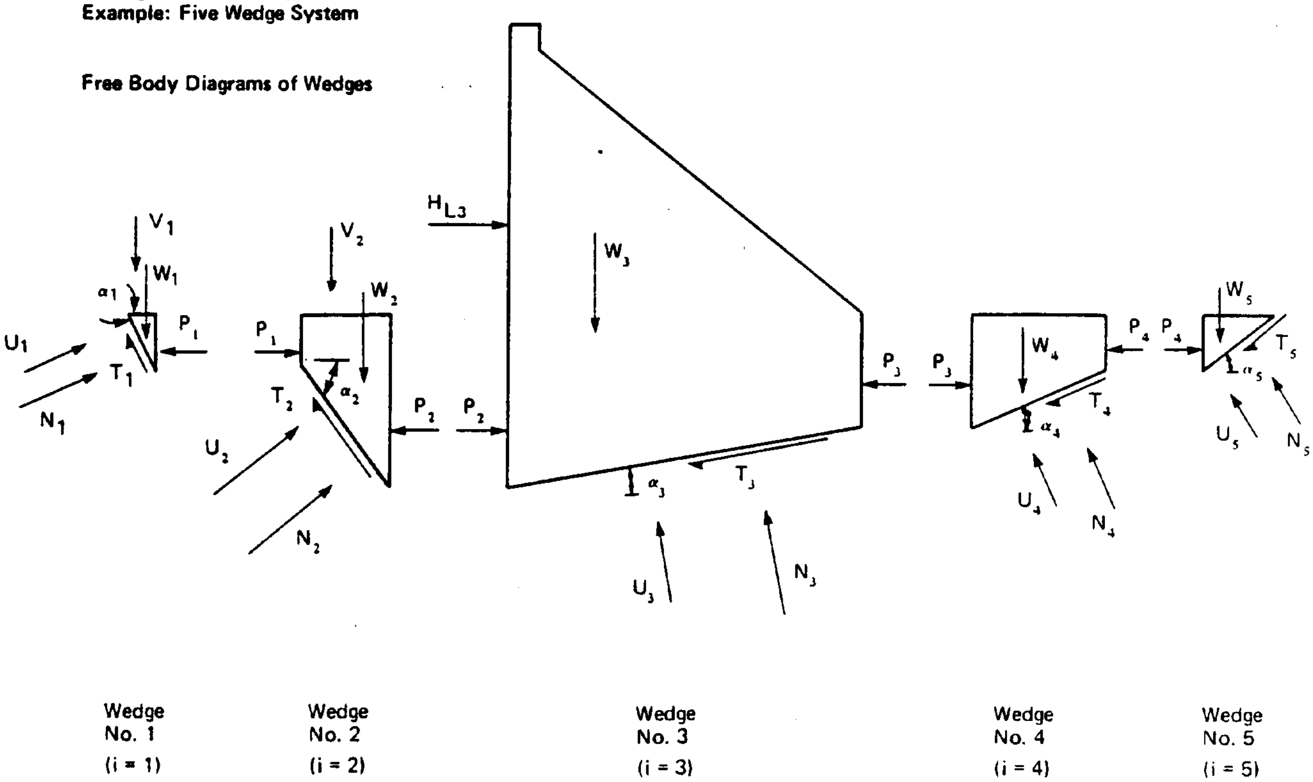
Example 2: Multiple Wedges
Determine the factor of safety against sliding for the following five wedge system



Geometry of Structure-Foundation System

Sliding Stability Analysis
Example: Five Wedge System

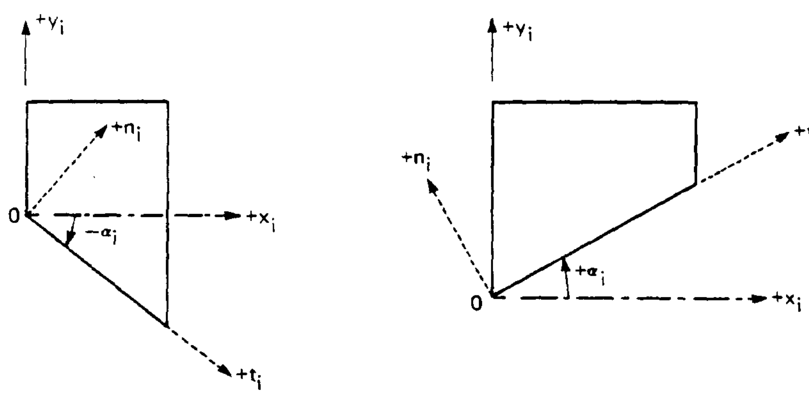
Free Body Diagrams of Wedges



Sliding Stability Analysis
Example: Five Wedge System

General Wedge Equation

$$P_{i-1} - P_i = \frac{[(W_i + V_i)\cos\alpha_i - U_i + (H_{Li} - H_{Ri})\sin\alpha_i]\dfrac{\tan\phi_i}{FS_i} - (H_{Li} - H_{Ri})\cos\alpha_i + (W_i + V_i)\sin\alpha_i + \dfrac{c_i}{FS_i}L_i}{(\cos\alpha_i - \sin\alpha_i\dfrac{\tan\phi_i}{FS_i})}$$



Sign Convention for General Equation

Wedge Forces for Trial Safety Factor of 1.5

<u>i = 1</u> $H_{Li} = H_{Ri} = 0$

$$\tan\phi_d = \frac{\tan\phi_1}{FS_1} = \frac{\tan 20}{1.5}$$

$$\phi_d = \tan^{-1}(0.243) = 13.64^o$$

$$\alpha_1 = -(45^o + \frac{\phi_d}{2}) = -51.82^o$$

$\sin(-51.82) = -0.786$

$\cos(-51.82) = 0.618$

$L_1 = 5/|\sin(-51.82)| = 5/0.786 = 6.36'$

[This orientation of the failure path is only true if the stratification and surface are horizontal

Sliding Stability Analysis
Example: Five Wedge System

$$W_1 = \frac{1}{2}(0.117)\,(5)*6.36\cos(-51.82) = 1.15^k\downarrow$$

$$V_1 = (25*.0625)\,6.36\cos(-51.82) = 6.14^k\downarrow$$

$\left.\right\}\ 7.29^k\downarrow$

$$U_1 = \frac{1}{2}(.0625)\,(25+30)\,6.36 = 10.93^k\nearrow$$

$$(P_o - P_1) = \frac{[7.29\,(0.618) - 10.93]\dfrac{\tan 20}{1.5} + 7.29\,(--0.786)}{[0.618 - (-0.786)\dfrac{\tan 20}{1.5}]} = -9.01^k$$

$$\boxed{(P_o - P_1) = \overleftarrow{9.01}^k}$$

<u>i = 2</u> $H_{L_2} = H_{R_2} = 0$

$$\tan\phi_d = \frac{\tan\phi_2}{FS_2} = \frac{\tan 30}{1.5} \qquad \phi_d = \tan^{-1}(0.385) = 21.05^o$$

$$\alpha_2 = -\left(45 + \frac{\phi_d}{2}\right) = -55.53^o$$

$\sin(-55.53) = -0.8244$ $\qquad$ $\cos(-55.53) = 0.566$

$L_2 = 10/\,|\sin(-55.53)| = 12.13'$

$$W_2 = 0.117\,(5)\,(12.13^x 0.566) + \frac{1}{2}(.122)\,(10)\,(12.13^x 0.566) = 8.20^k\downarrow$$

$$V_2 = (25^x.0625)\,(12.13^x 0.566) = 10.73^k\downarrow$$

$$U_2 = \frac{1}{2}(0.0625)\,(30+40)\,(12.13) = 26.53^k\nearrow$$

Sliding Stability Analysis
Example: Five Wedge System

$$(P_1 - P_2) = \frac{[18.93\,(0.566) - 26.53]\dfrac{\tan 30}{1.5} + 18.93\,(-.8244)}{[0.566 - (-0.8244)\dfrac{\tan 30}{1.5}]} = -24.56^k$$

$$\boxed{(P_1 - P_2) = \overleftarrow{24.56}^k}$$

i = 3 $\alpha_3 = 9.5^o$ $L_3 = 5/\sin 9.5 = 30.3'$

$H_{L_3} = \frac{1}{2}(0.0625)\,(25)^2 = \overrightarrow{19.53}^k$ $H_{R_3} = 0$

$U_3 = \frac{1}{2}\,(0.0625)\,(40+10)\,(30.3) = 47.33^k \nwarrow$

$W_3 = 122.4^k \downarrow$

$\sin 9.5^o = 0.165$ $\cos 9.5 = 0.986$

$$(P_2 - P_3) = \frac{[122.4\,(.986) - 44.11]\dfrac{\tan 30}{1.5} - 19.53\,(0.986) + 122.4\,(.165)}{(.986 - .165 \times \dfrac{\tan 30}{1.5})} = 32.97^k$$

$$\boxed{(P_2 - P_3) = \overrightarrow{32.97}^k}$$

i = 4 $H_{L_4} = H_{R_4} = V_4 = 0$

$\tan \phi_d = \frac{\tan \phi_4}{FS_4} = \frac{\tan 30^o}{1.5}$ $\phi_d = \tan^{-1}(0.385) = 21.05^o$

$\alpha_4 = 45 - \frac{1}{2}\phi_d = 34.475^o$

$\sin (34.475) = 0.566$ $\cos (34.475^o) = 0.824$

Sliding Stability Analysis
Example: Five Wedge System

$L_4 = 5/\sin 34.475 = 8.83'$

$$W_4 = (0.132)\ (5)\ (8.83^{\times}.824) + \frac{1}{2}(0.122)\ (5)\ (8.83^{\times}0.824) = 7.02^k \downarrow$$

$$U_4 = \frac{1}{2}(0.0625)\ (5+10)\ (8.83) = 4.14^k \nwarrow$$

$$(P_3 - P_4) = \frac{[7.02\ (.824) - 4.14]\ \frac{\tan 30}{1.5} + 7.02\ (.566)}{[0.824 - 0.566\ \frac{\tan 30}{1.5}]} = 7.59^k$$

$$\boxed{(P_3 - P_4) = \vec{7.59}^k}$$

<u>i = 5</u> $H_{L_5} = H_{R_5} = V_5 = 0$

$$\tan \phi_d = \frac{\tan \phi_5}{FS_5} = \frac{\tan 40}{1.5} \qquad \phi_d = \tan^{-1}(0.559) = 29.22^o$$

$$\alpha_5 = (45 - \frac{1}{2}\phi_d) = 30.38 \qquad \sin 30.38 = 0.5058 \qquad \cos 30.38 = 0.8626$$

$L_5 = 5/\sin 30.38 = 9.89'$

$$W_5 = \frac{1}{2}(0.132)\ 5\ (9.89^{\times}0.8626) = 2.82^k \downarrow$$

$$U_5 = \frac{1}{2}(0.0625)\ (5)\ (9.89) = 1.545^k \nwarrow$$

$$(P_4 - P_5) = \frac{[2.82^{\times}.863 - 1.54]\ \frac{\tan 40}{1.5} + 2.82^{\times}.506}{[.863 - .506\ \frac{\tan 40}{1.5}]} = 3.32^k$$

$$\boxed{(P_4 - P_5) = \overrightarrow{3.32}^k}$$

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Sliding Stability Analysis
Example: Five Wedge System

Summary: Wedge Forces for Trial Safety Factors

FS = 1.5

i	α_i	L_i	H_{Li}	H_{Ri}	V_i	W_i	U_i	$(P_{i-1} - P_i)$
1	−51.82	6.36	0	0	6.14	1.15	10.93	−9.01
2	−55.53	12.13	0	0	10.73	8.20	26.53	−24.56
3	9.5	30.3	19.53	0	0	122.4	47.33	32.97
4	34.47	8.83	0	0	0	7.02	4.14	7.59
5	30.38	9.89	0	0	0	2.82	1.54	3.32
							ΔP_R =	10.31

FS = 2.5

i	α_i	L_i	H_{Li}	H_{Ri}	V_i	W_i	U_i	$(P_{i-1} - P_i)$
1	−49.14	6.61	0	0	6.75	1.27	11.36	−9.10
2	−51.5	12.78.	0	0	12.43	9.50	27.95	−25.48
3	9.5	30.3	19.53	0	0	122.4	47.33	19.65
4	38.5	8.0	0	0	0	6.06	3.76	6.26
5	35.72	8.56	0	0	0	2.29	1.34	2.45
							ΔP_R =	-6.20

FS = 2.0

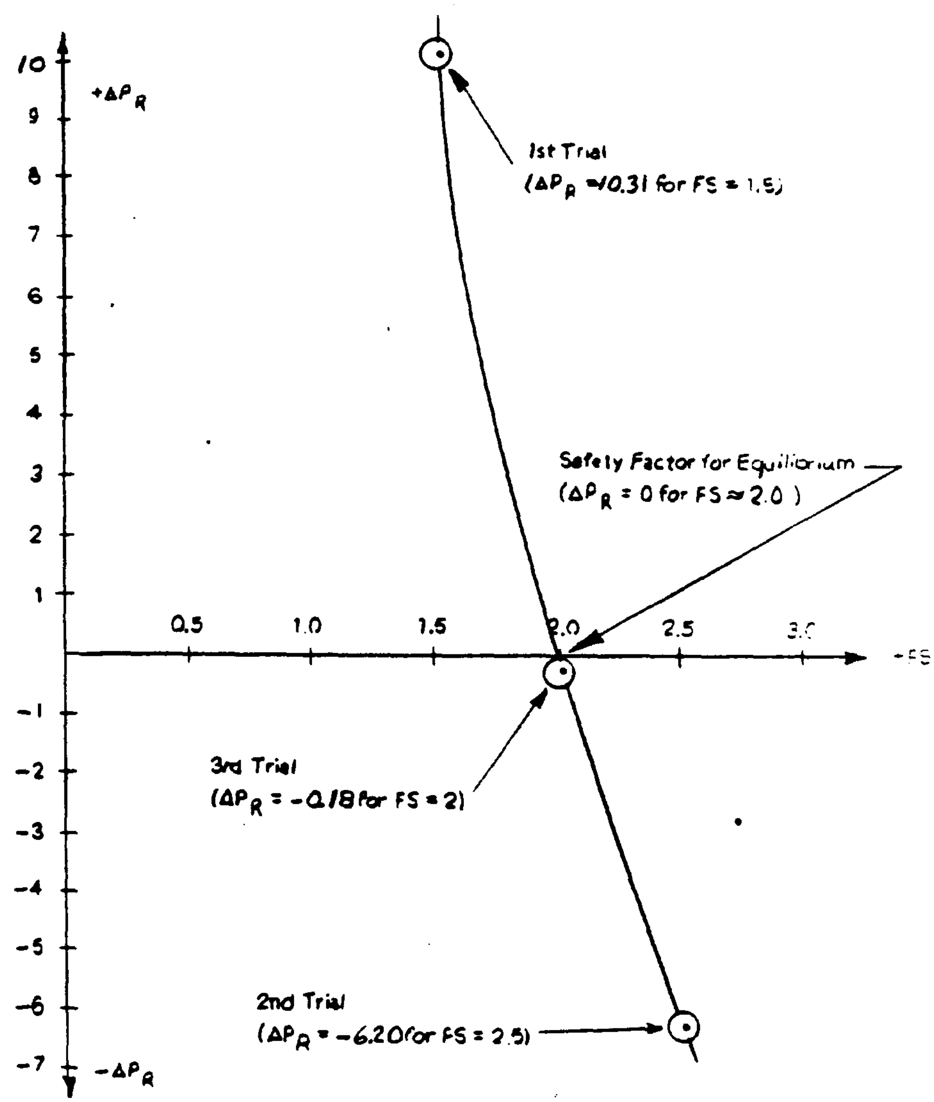
i	α_i	L_i	H_{Li}	H_{Ri}	V_i	W_i	U_i	$(P_{i-1} - P_i)$
1	−50.16	6.51	0	0	6.52	1.22	11.19	−9.06
2	−53.05	12.51	0	0	11.73	8.97	27.37	−25.13
3	9.5	30.3	19.53	0	0	122.4	47.33	24.53
4	36.95	8.33	0	0	0	6.43	3.9	6.73
5	33.62	9.03	0	0	0	2.48	1.41	2.75
							ΔP_R =	-0.18

Sliding Stability Analysis
Example: Five Wedge System

Graphical Solution for Safety Factor

The safety factor for sliding equilibrium of the five wedge system is determine from:

$$\sum_{i=1}^{5} (P_{i-1} - P_i) = \Delta P_R \begin{cases} \Delta P_R = 0 & \text{Safety factor for equilibrium} \\ \Delta P_R \neq 0 & \text{For trial safety factors} \end{cases}$$



ALTERNATE METHOD OF ANALYSIS

1. Definition of Factor of Safety. This sliding stability criteria is based upon presently acceptable geotechnical principles with respect to shearing resistance of soils and rock, and applies the factor of safety to the least known conditions affecting sliding stability; that is, the material strength parameters. The factor of safety is related to the required shear stress and available shear strength according to Equation 1A:

$$\tau = \frac{\tau_a}{FS} \tag{1A}$$

where

τ = the required shear stress for safe stability
τ_a = the available shear strength
FS = the factor of safety

The most accepted criteria for defining the available shear strength (τ_a) of a given material is the Mohr-Coulomb failure criteria. Equation 1A may be rewritten as:

$$\tau = (c + \sigma \tan \phi)/FS \tag{2A}$$

in which

c = the cohesion intercept
σ = the normal stress on the shear plane
ϕ = the angle of internal friction

The ratio $\frac{\tau a}{FS}$ can be considered as the degree of shear mobilization.

2. Solutions for Factor of Safety. The following equations for evaluating sliding stability were developed from the definition of FS and the assumption discussed in paragraph one above. The equations provide FS solutions for both single and multiple-plane failure surfaces, using any number of blocks or wedges.

a. Notation:

c = the cohesion intercept

U = the uplift force acting under a wedge on the critical potential failure plane = uplift pressure x area of critical potential failure plane

A = the area of the critical potential failure plane

V = all applied vertical forces (body and surcharge) acting on an individual wedge

H = all applied horizontal forces acting on an individual wedge

α = the angle between the inclined plane of the critical potential failure surface and the horizontal ($\alpha > 0$ for upslope sliding; $\alpha < 0$ for downslope sliding)

ϕ = the angle of internal friction along the critical potential failure plane considered

i = the subscript associated with planar segments along the critical potential failure surface

N = the number of wedges in the failure mechanism or number of planes making up the critical potential failure surface

b. Case 1: Single-Plane Failure Surface. Figure 3-1 shows a graphical representation of a single-plane failure mode. Here the critical potential failure surface is defined by a single plane at the interface between the structure and foundation material with no embedment. Equation 3A provides a direct solution for FS for inclined failure planes.

$$FS = \frac{cA + (V \cos \alpha - U + H \sin \alpha) \tan \phi}{H \cos \alpha - V \sin \alpha} \qquad (3A)$$

For the case where the critical potential failure surface can be defined as a horizontal plane ($\alpha = 0$), Equation 3A reduces to Equation 4A:

$$FS = \frac{cA + (V - U) \tan \phi}{H} \qquad (4A)$$

c. Case 2: Multiple-Plane Failure Surface. This general case is applicable to situations where the structure is embedded and/or where the critical potential failure surface is defined by two or more weak planes. The solution for FS is obtained from Equation 5A:

$$FS = \frac{\sum_{i=1}^{N} \frac{c_i A_i \cos \alpha_i + (V_i - U_i \cos \alpha_i) \tan \phi_i}{n_{\alpha i}}}{\sum_{i=1}^{N} (H_i - V_i \tan \alpha_i)} \quad (5A)$$

where

$$n_{\alpha_i} = \frac{1 - \frac{\tan \phi_i \tan \alpha_i}{FS}}{1 + \tan^2 \alpha_i}$$

Figure 3-2 shows a graphical representation of a multiple (in its simplest form, two planes) plane failure mode.

3. Use of Equations and Limitations of Analytic Techniques.

a. Case One: Single-Plane Failure Surface. The solution for the factor of safety is explicit by use of Equations 3A and 4A. These equations satisfy both vertical and horizontal static equilibrium. However, the user should be aware that in cases for which $\alpha > 0$ (upslope sliding) and where $H/V \lesssim \tan \alpha$, Equation 3A results in a $FS = \infty$ or a negative FS; in these cases, solutions for FS do not have meaning.

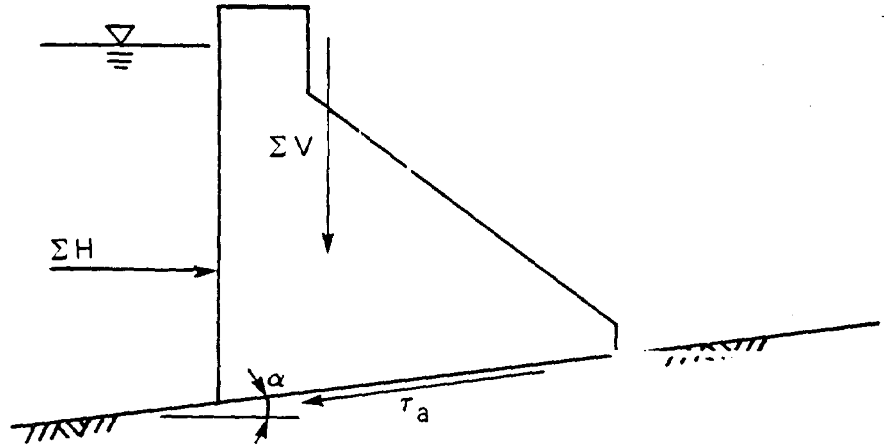
b. Case Two: Multiple-Plane Failure Surface.

(1) Equation 5A is implicit in FS (except when $\phi = 0$ or $\alpha = 0$) since n_α is a function of FS. Therefore, the mathematical solution of Equation 5A requires an iteration procedure. The iteration procedure requires that an initial estimate of FS be inserted into the n_α term and a FS calculated. The calculated FS is then inserted into the n_α term and the process is repeated until the calculated FS converges with the inserted FS. Generally, convergence occurs within four to five iterations. The iteration process can be performed manually or the equation can be easily programmed for a programmable calculator. To facilitate hand solution, a plot of n_α versus α for values of $\tan \phi / FS$ is given in Figure 3-3.

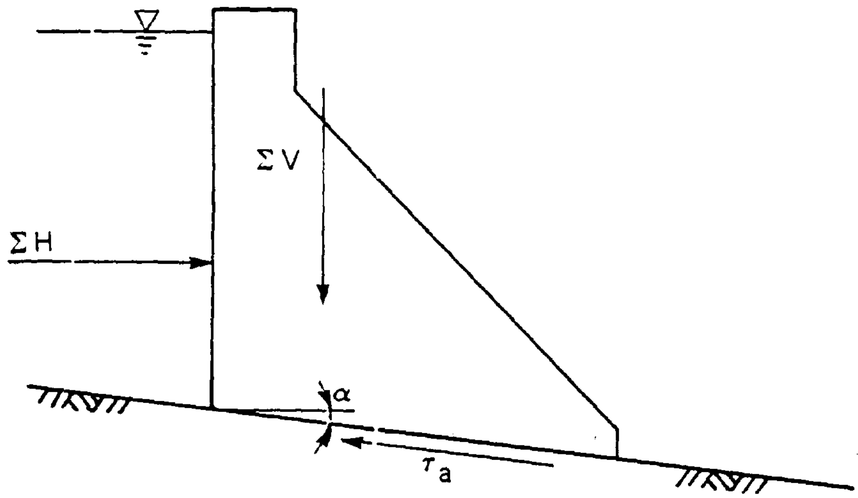
(2) Equation 5A is similar to the generalized method of slices for sliding stability criteria. However, in order to develop a simple analytic technique suitable for routine use, the vertical side forces due to impending

motion of the wedges between slices were assumed to be zero. Therefore, although the equation satisfies complete horizontal static equilibrium, complete vertical equilibrium is in general not satisfied. The FS computed from Equation 5A will be slightly lower than the FS computed from the more complicated techniques which completely satisfy both vertical and horizontal static equilibrium.

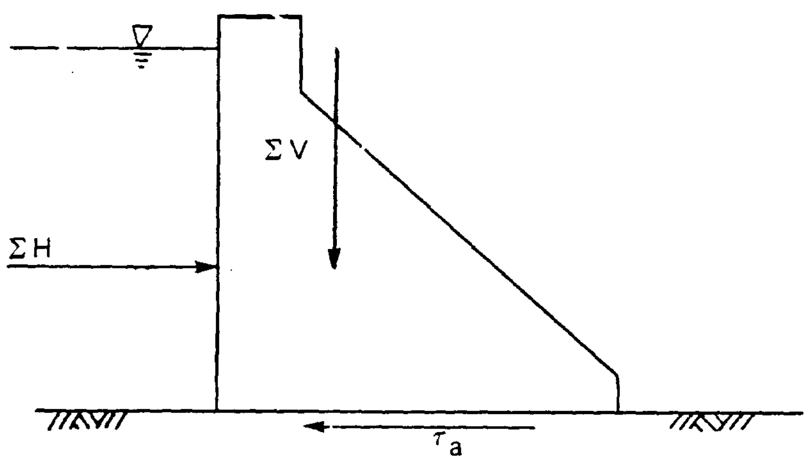
(3) The user should be aware that Equation 5A will yield identical solutions for FS with the methods described in the main body of this ETL. The governing wedge equation (equation seven), together with the boundary conditions (equations three and four) to have the system of wedges act as an integral failure mechanism, is mathematically equivalent to Equation 5A. The user may find the more convenient method to be a function of the design situation. Since solutions for FS by these two methods of analysis are identical, and since the mathematical approach is quite different, one can effectively be used as a check on the other.



a. Upslope Sliding, $\alpha > 0$



b. Downslope Sliding, $\alpha < 0$



c. Horizontal Sliding, $\alpha = 0$

Figure 3-1. Single Plane Failure Mode

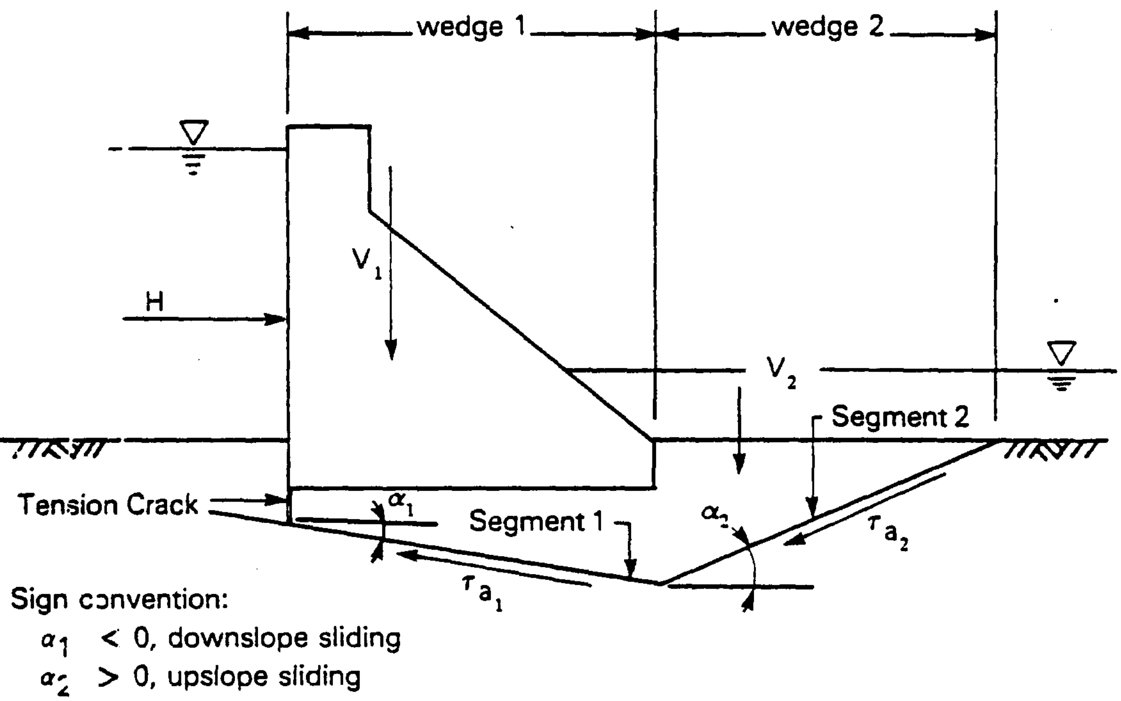


Figure 3-2. Multiple Plane Failure Mode in the Simplist Form of Two Planes.

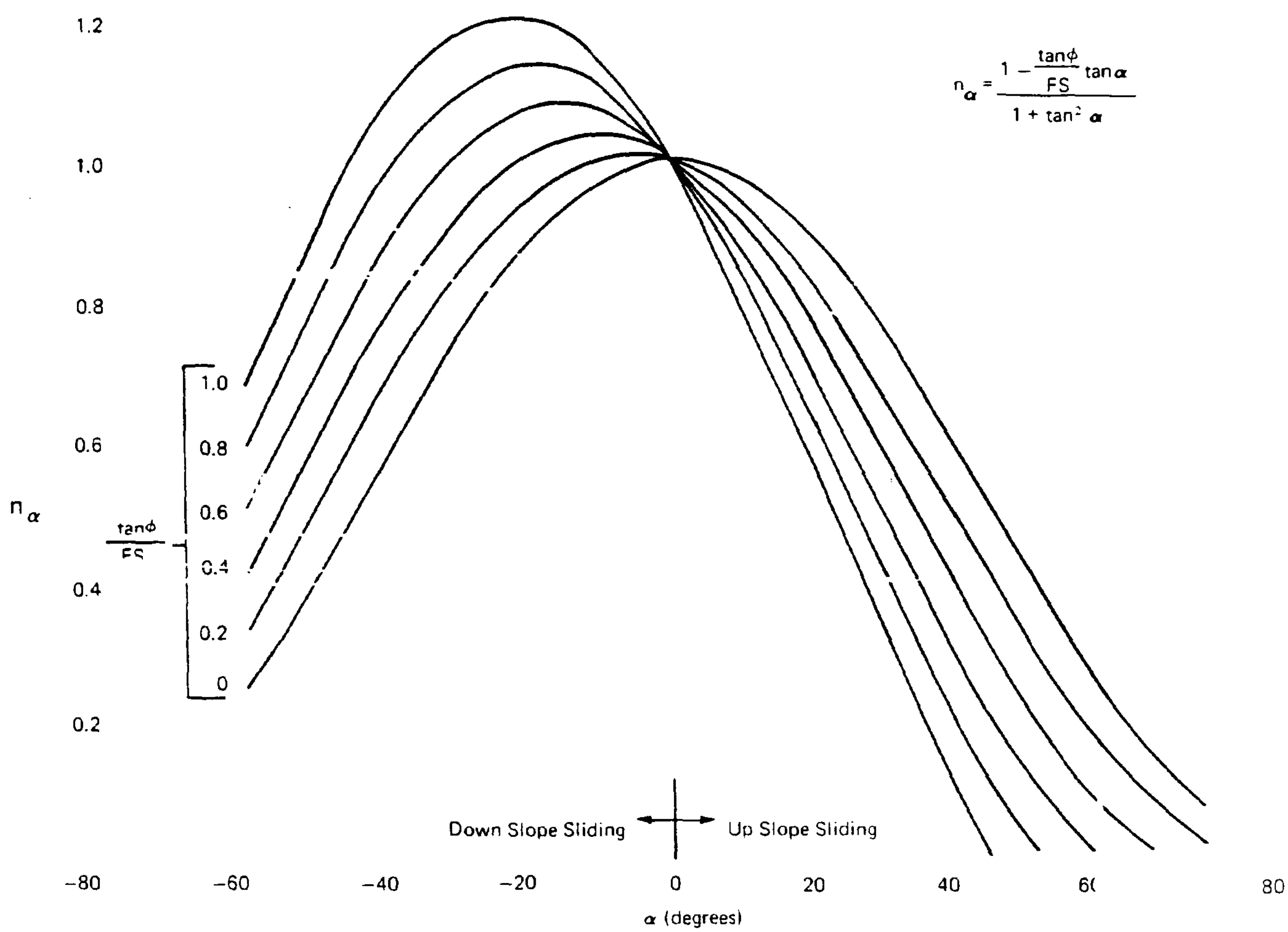


Figure 3-3. Plot of n_α and α for Values of $\tan\phi$/FS